COMPLETE CLASSIFICATION OF 1+1 GRAVITY SOLUTIONS

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A classification of the maximally extended solutions for 1+1 gravity models (comprising e.g. generalized dilaton gravity as well as models with non-trivial torsion) is presented. No restrictions are placed on the topology of the arising solutions, and indeed it is found that for generic models solutions on non-compact surfaces of arbitrary genus with an arbitrary non-zero number of holes can be obtained. The moduli space of classical solutions (solutions of the field equations with fixed topology modulo gauge transformations) is parametrized explicitly.

1 Introduction

A feature shared by practically all matterless 2D gravity models is the presence of a Killing symmetry. The models of this type comprise all 2D dilaton gravity theories,[1]

$$
L[g, \Phi] = \int_{\mathcal{M}} \mathrm{d}^2 x \sqrt{|\det g|} \big[U(\Phi)R + V(\Phi) + W(\Phi)\partial_\mu \Phi \partial^\mu \Phi \big] , \tag{1}
$$

thus including e.g. spherically reduced gravity or R^2-gravity, but also generalizations with non-trivial torsion, not contained in (1).[2] The classification of solutions splits naturally into two parts: local considerations, i.e. solving the field equations for the metric and other fields on some local patch, and global ones, like finding the maximal extensions, determining their topology and causal structure.

2 Local Issues

Due to the Killing symmetry the metric may locally always be brought into Schwarzschild (SS) or equally well into Eddington-Finkelstein (EF) form[2,3]

$$
g^{SS} = h(r)\mathrm{d}t^2 - \frac{1}{h(r)}\mathrm{d}r^2 , \qquad g^{EF} = 2\mathrm{d}r\mathrm{d}v + h(r)\mathrm{d}v^2 , \tag{2}
$$

the function $h(r)$ being the same in both cases. In contrast to the SS-form, the EF-form covers smoothly also the Killing horizons (zeros of $h(r)$), allowing for a simple extension algorithm (see below). Other fields of physical relevance like the dilaton Φ also depend on r only. Thus, locally the classification amounts to specifying the possible functions $h(r)$ and $\Phi(r)$. Solving the field equations for these functions is relatively straightforward and leads, for a fixed Lagrangian (1), to a one-parameter family $h = h_M(r)$, where M can often be given a physical interpretation as black-hole mass.

3   Global Issues

Unless $h(r)$ does not have zeros, the EF-patches (2) are incomplete and thus have to be extended. Usually, for a solution to be maximally extended one expects that geodesics should be complete or, if not, the curvature or some other physical field like the dilaton Φ should blow up along them. These requirements are not only highly reasonable physically, solutions of this type also allow for a concise classification. Unfortunately, there are other types of inextendibility, namely if the extension candidate
— would not be Hausdorff (Taub-NUT spaces),
— would not be smooth (conical singularity).
Especially solutions of the latter type abound: Just take *any* manifold, cut out a point and take a covering manifold thereof. Clearly, the removed point cannot be inserted any longer, since it would lead to a conical singularity (branch point). Several popular kink metrics given in the literature, but also recently discovered continuous families of kinks are exactly of this type.[4] In what follows, however, we will disregard such solutions.

The extensions are best dealt with in two steps, constructing the simply connected *universal coverings* (u. cov.) first. To this end one exploits the symmetry of the metric under inversion of the Killing parameter, $t \leftrightarrow -t$ in SS-coordinates. In EF-coordinates this transformation reads

$$r \mapsto r, \qquad v \mapsto -v - 2 \int \frac{\mathrm{d}r}{h(r)}, \tag{3}$$

valid between two successive zeros of $h(r)$. Using (3) as transition map for overlapping EF-patches and adding special maps for the neighbourhood of bifurcation points, one obtains an atlas for the u. cov.[2] This construction turns out to be unique for a given $h(r)$. The corresponding multiply connected solutions can all be obtained in a second step, by factoring this u. cov. by adequately acting subgroups $\mathcal{H}$ of the symmetry group $\mathcal{G}$. Moreover, the inequivalent factor spaces are in one-to-one correspondence with the conjugacy classes of subgroups $\mathcal{H} \leq \mathcal{G}$.

Applying the above program to a given metric (2), one finds the global properties of the arising solutions to depend only on the number and degree of the zeros of $h(r)$. Let us summarize the results for space- and time-orientable solutions with merely simple zeros of $h(r)$:[2]

- there is a unique simply connected solution (universal covering), which is topologically a disc,

- for no resp. two simple zeros of $h(r)$ there occur additionally cylinders,

- for ≥ 3 simple zeros there occur solutions of arbitrary non-compact topology,

- the number of additional continuous parameters (besides M) equals the rank of π_1(spacetime manifold $\mathcal{M}$).

Note, however, that even within *one* model, i.e. for a fixed Lagrangian, the number of zeros of $h(r) \equiv h_M(r)$ may change with the parameter M (see e.g. Fig. 1). Especially, for a "generic" choice of U, V, W in the action (1), the occurrence of

$L[g,\Phi]$

no further solutions

to be continued

etc.

to be continued

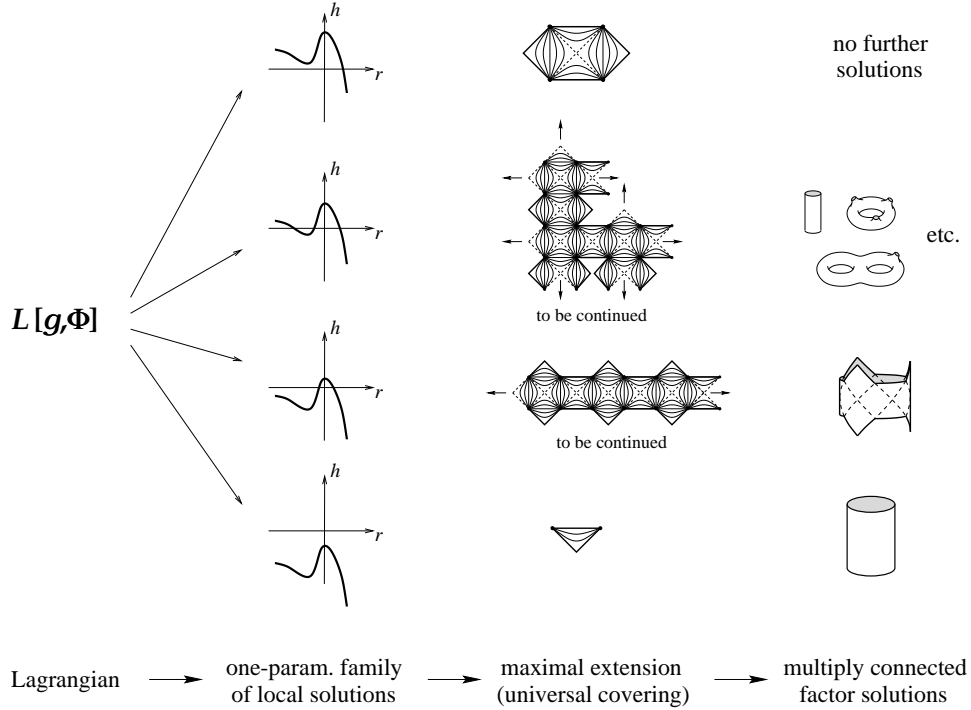| Lagrangian | $\longrightarrow$ | one-param. family of local solutions | $\longrightarrow$ | maximal extension (universal covering) | $\longrightarrow$ | multiply connected factor solutions |

Figure 1: Classification. A typical example.

three or more zeros is the rule, resulting in solutions of all non-compact topologies. For a fixed topology of $\mathcal{M}$ the moduli space, if non-empty, has dimension $\pi_1(\mathcal{M})+1$.

Acknowledgments

References

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